M. Sc. in Mathematics
Course Layout

Subject Code	Paper name	Credits	L-T-P
	Semester I		
MTH-PG-C101	Analysis-I	4	4-0-0
MTH-PG-C102	Linear Algebra	4	4-0-0
MTH-PG-C103	Combinatorics and Elementary Number Theory	4	4-0-0
MTH-PG-C104	Differential Equations	4	4-0-0
	Semester II		
MTH-PG-C201	Analysis II	4	4-0-0
MTH-PG-C202	Complex Function Theory	4	4-0-0
MTH-PG-C203	Algebra	4	4-0-0
MTH-PG-O204	(A) Numerical Analysis (B) Cryptography (Open Elective)	4	4-0-0
	Semester III		
MTH-PG-C301	Topology	4	4-0-0
MTH-PG-C302	Field Theory	4	4-0-0
MTH-PG-C303	Measure Theory	4	4-0-0
MTH-PG-O304	(A) Optimization Technique (B) Nonlinear Dynamics (Open Elective)	4	4-0-0
	Semester IV		
	Elective-1	4	4-0-0
	Elective-2	4	4-0-0
	Elective-3	4	4-0-0
	Elective-4	4	4-0-0
	Project Work*	4	0-0-4
Total Credits for M.Sc. 64			

Open Elective: Papers which shall be opted from other departments
Elective: Papers which shall be opted from the department
*Students having good SGPA are eligible to take a project of 4 credits in lieu of one elective paper in final semester, subject to the approval of the department.

Departmental Electives:
1. Algebraic Topology
2. Commutative Algebra
3. Algebraic Geometry
4. Differential Geometry
5. Operator theory
6. Functional Analysis

MTH-PG-C101: ANALYSIS-I

Credit: 4

Unit I: Real Numbers

Relations and functions, Finite and infinite sets, countable and uncountable sets, least upper bound property, the field of real numbers, Archimedean property, density of rational numbers, existence of n^{th} root of positive real numbers, exponential and logarithm, the extended real number system.

Unit II: Numerical Sequences and Series

Numerical sequences and their convergence, bounded sequences, Cauchy sequences, construction of real numbers using Cauchy sequences; limit supremum and limit infimum, Bolzano-Weierstrass' threorem for sequences of real number, series of nonnegative terms, the number e, tests of convergence of series, power series, absolute convergence, addition and multiplication of series, rearrangements (statement only).

Unit III: Topology of R^n

Euclidean spaces, open and closed sets, limit points, interior points, compact subsets of R^n, nested interval theorem, Heine-Borel theorem, and Bolzano-Weierstrass' threorem.

Limits of functions, continuous functions, continuity and compactness, uniform continuity, connected sets, connected subsets of real numbers, continuity and connectedness, intermediate value theorem; discontinuities and their classifications, monotonic functions, infinite limits and limits at infinity.

Unit IV: Differentiation & Integration

Differentiation of real-valued functions and its elementary properties; mean value theorem; Taylor's theorem; elementary properties of Riemann integral, Fundamental theorem of Calculus, mean value theorem, convergence of improper integrals.

Textbooks:

1. Rudin, W. (2013) *Principles of Mathematical Analysis* (3rd Edition), Tata McGraw Hill Education.
2. Rotman, J.J. (2002) *An Introduction to the Theory of Groups* (4th edition) Allyn and Bacon, Inc., Boston.

Reference books:

1. Apostol, T. (2000) *Mathematical Analysis* (2nd edition) Narosa Book Distributers Pvt. Ltd.
2. Bartle, R.G. and Sherbert D. R. (2000) *Introduction to Real Analysis* (3rd edition) John Wiley & Sons, Inc., New York.
3. Fraleigh, J. B. (2002) *A First Course in Abstract* Algebra (4th edition) Narosa Publishing House, New Delhi.
4. Gallian, J. A. (1999) *Contemporary Abstract Algebra* (4th edition) –Narosa Publishing House, New Delhi.

MTH-PG-C102: LINEAR ALGEBRA

Credit: 4

Unit I: Vector Space

Vector spaces, linear independence; linear transformations, matrix representation of a linear transformation; isomorphism between the algebra of linear transformations and that of matrices;

Unit II: Eigenvalues and Eigenvectors

Similarity of matrices and linear transformations; trace of matrices and linear transformations, characteristic roots and characteristic vectors, characteristic polynomials, relation between characteristic polynomial and minimal polynomial; Cayley-Hamilton theorem (statement and illustrations only); diagonalizability, necessary and sufficient condition for diagonalizability;

Unit III: Canonical Forms

Projections and their relation with direct sum decomposition of vector spaces; invariant subspaces; primary decomposition theorem, cyclic subspaces; companion matrices; a proof of Cayley-Hamilton theorem; triangulability; canonical forms of nilpotent transformations; Jordan canonical forms; rational canonical forms.

Unit IV: Inner Product Spaces

Inner product spaces, properties of inner products and norms, Cauchy-Schwarz inequality; orthogonality and orthogonal complements, orthonormal basis, Gram-Schmidt process; adjoint of a linear transformation; Hermitian, unitary and normal transformations and their diagonalizations.

Textbooks:

1. Hoffman, K., Kunze, R. (2000) *Linear Algebra* (2^{nd} edition) Prentice Hall of India Pvt. Ltd., New Delhi.
2. Bhattacharya, P. B. Jain, S. K. and Nagpal, S. R. (2000) *First Course in Linear Algebra*, Wiley Eastern Ltd., New Delhi.

Reference books:

1. Herstein, I. N. (2003) *Topics in Algebra* (4^{th} edition), Wiley Eastern Limited, New Delhi.
2. Shilov, G. E. (1998) *Linear Algebra*, Prentice Hall Inc.
3. Halmos, P. R. (1965) *Finite Dimensional Vector Spaces*, D.Van Nostrand Company Inc.
4. Finkbeiner, D. T. (2011) *Introduction to Matrices and Linear Transformations* (3^{rd} edition) Dover Publications.
5. Kumaresan,S. (2001) *Linear Algebra: A Geometric Approach*, Prentice-Hall of India Pvt. Ltd., New Delhi.

MTH-PG-C103: COMBINATORICS AND ELEMENTARY NUMBER THEORY

Credit: 4

Unit I: Elementary Combinatorics
Divisibility; Euclidean algorithm; primes; congruences; Fermat's theorem, Euler's theorem and Wilson's theorem; Fermat's quotients and their elementary consequences; solutions of congruences; Chinese remainder theorem; Euler's phi-function.

Unit II: Congruences
Congruence modulo powers of prime; primitive roots and their existence; quadratic residues; Legendre symbol, Gauss' lemma about Legendre symbol; quadratic reciprocity law; proofs of various formulations; Jacobi symbol.

Unit III: Diophantine Equations
Solutions of $ax + by = c$, $x^2 + y^2 = z^2$, $x^4 + y^4 = z^2$; properties of Pythagorean triples; sums of two, four and five squares; assorted examples of diophantine equations.

Unit IV: Generating Functions and Recurrence Relations
Generating Function Models, Calculating coefficient of generating functions, Partitions, Exponential Generating Functions, A Summation Method. Recurrence Relations: Recurrence Relation Models, Divide and conquer Relations, Solution of Linear, Recurrence Relations, Solution of Inhomogeneous Recurrence Relations, Solutions with Generating Functions.

Textbooks:

1. Niven, I., Zuckerman, H. S. and Montegomery, H. L. (2003) *An Introduction to the Theory of Numbers* (6th edition) John Wiley and sons, Inc., New York.
2. Burton, D. M. (2002) *Elementary Number Theory* (4th edition) Universal Book Stall, New Delhi.
3. Balakrishnan, V. K. (1994) *Schaum's Outline of Theory and Problems of Combinatorics Including Concepts of Graph Theory*, Schaum's Outline.
4. Balakrishnan,V. K. (1996) *Introductory Discrete Mathematics*, Dover Publications.

Reference books:

1. Dickson, L. E. (1971) *History of the Theory of Numbers* (Vol. II, Diophantine Analysis) Chelsea Publishing Company, New York.
2. Hardy, G.H. and Wright, E. M.(1998) *An Introduction to the Theory of Numbers* (6th edition),The English Language Society and Oxford University Press.
3. Niven, I. and. Zuckerman, H. S. (1993) *An Introduction to the Theory of Numbers* (3rd edition), Wiley Eastern Ltd., New Delhi.

MTH-PG-C104: DIFFERENTIAL EQUATIONS

Credit: 4

Unit I: Initial Value Problems

Existence and uniqueness of solutions of IVP: method of successive approximations, system of first order approximations, Picards theorem, Continuous dependence of the solution on initial data, general theory of system of first order equations. Linear systems: Homogenous and nonhomogenous linear systems with constant coefficients.

Unit II: Series Solution

Power series solution, second order equations, ordinary points, regular points and singular points, special functions, Hermite polynomials, Chebychev polynomials, Legendre polynomials, Bessel functions, Gamma functions

Unit III: Boundary Value Problem

Boundary value problem, Green function, Sturm-Liouville Theory, eigenvalues and eigenfunctions, qualitative properties of solutions, Sturm comparison theorem, Sturm separation theorem

Unit IV: Partial Differential Equations

First order equations, Cauchy Kowlewski theorem, classification of second order PDE, canonical form second order linear equations with constant co-efficients, method of separation of variables. Characteristics and uniqueness theorem for hyperbolic theorems with initial boundary conditions. Elliptic and Parabolic partial differential equations

Text Book:

1. Ross, S. L. (1984) *Differential Equations* (3rd edition), John Wiley & Sons.
2. Coddington, E. A. *An Introduction to Ordinary Differential Equations*, Prentice-Hall.
3. Sneddon, I.N. (1957) *Elements of Partial Differential Equations*, McGraw Hill.

References:

1. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, McGraw HillEducation.
2. Fritz John (1982) *Partial Differential Equations*, Springer-Verlag, New York Inc.
3. Coddington, E. A. and Levinson, N. (1955) *Theory of Ordinary Differential equations*, TMH Education.
4. Cronin, J. (1994) *Differential Equations: Introduction and Qualitative Theory*, Marcel Dekker.
5. Hirsch, M.W. Smale, S. and Devaney, R.L. (2004) *Differential Equations, Dynamical Systems and an Introduction to Chaos*, Elsevier.

MTH-PG-C201: ANALYSIS II

Credit: 4

Unit I: Sequences & Functions

Sequences of functions, pointwise and uniform convergence, uniform convergence and continuity, uniform convergence and integration, uniform convergence and differentiation, Weierstrass' approximation theorem.

Unit II: Differentiability of Functions of Several Variables

Directional derivatives and differentiability of functions of several variables and their interrelationship, chain rule, mean value theorem; higher order partial derivatives, equality of mixed partial derivatives, Taylor's theorem.

Unit III: Inverse & Implicit Function Theorem

Injective mapping theorem, surjective mapping theorem, inverse function theorem and implicit function theorem of functions of two and three (for analogy) variables, extremum problems with and without constraints of functions of two and three (for analogy) variables.

Unit IV: Multiple Integrals

Multiple integrals, repeated integrals, interchange of order of integrations, change of variable theorem, mean-value theorems for multiple integrals, line integral and Green's theorem, Convergence of improper integrals.

Textbooks:

1. Rudin, W. (2013) *Principles of Mathematical Analysis* (3rd Edition), Tata McGraw Hill Education.
2. Apostol, T. (2000) *Mathematical Analysis* (2nd edition) Narosa Book Distributers Pvt. Ltd.
3. Bartle, R.G. (1994)*The Elements of Real Analysis* (3rd edition), Wiley International Edition.

Reference books:

1. Buck, R. C. &Buck, E. F. (1999) *Advanced Calculus* (4th Edition), McGraw Hill, New York.
2. Simmons, G. F.(2003) *Introduction to Topology and Modern Analysis* (4th edition), McGraw Hill.
3. Bartle, R. G. and Sherbert, D. R. (2000) *Introduction to Real Analysis* (3rd edition), John Wiley & Sons, Inc., New York.

MTH-PG-C202: COMPLEX FUNCTION THEORY

Credit: 4

Unit I: Holomorphic Functions

Holomorphic Functions, Cauchy-Riemman equations and its applications, Formal power series, radius of convergence of power series, exponential, cosine and sine, logarithm functions introduced as power series, their elementary properties.

Unit II: Complex Integration

Integration of complex-valued functions and differential 1-forms along a piecewise differentiable path, primitive, local primitive and primitive along a path of a differential 1-form, Index of a closed path, Cauchy's theorem for convex regions.

Cauchy's integral formula, Taylor's expansion of holomorphic functions, Cauchy's estimate; Liouville's theorem; fundamental theorem of algebra; zeros of an analytic function and related results; maximum modulus theorem; Schwarz' lemma.

Unit III: Singularities and Residues

Laurent expansion of a holomorphic function in an annulus, singularities of a function, removable singularities, poles and essential singularities; extended plane and stereographic projection, residues, calculus of residues; evaluation of definite integrals; argument principle; Rouche's Theorem.

Unit IV: Conformal Mapping

Complex form of equations of straight lines, half planes, circles, etc., analytic (holomorphic) function as mappings; conformal maps; Mobius transformation; cross ratio; symmetry and orientation principle; examples of images of regions under elementary analytic function.

Textbooks:

1. Sarason, D. (2008) *Complex Function Theory, Texts and Readings in Mathematics*, Hindustan Book Agency, New Delhi.

Reference books:

1. Ahlfors, L. V. (1990) *Complex Analysis* (2nd Edition), McGraw-Hill International Student Edition.
2. Conway, J. B. (2000) *Functions of one complex variable*, Springer International Student edition, Narosa Publishing House, New Delhi.
3. Churchill, R. V. (1996) *Complex Variables and applications*, McGraw-Hill.
4. Copson, E. T. (1995) *An Introduction to the Theory of functions of a complex Variable,* Oxford University Press.
5. Shastri, A. R. (2003) *An Introduction To Complex Analysis*, Macmillan India Ltd.

MTH-PG-C203: ALGEBRA

Credit: 4

Unit I: Basic Concepts of Groups

A brief review of groups, their elementary properties and examples, subgroups, cyclic groups, homomorphism of groups and Lagrange's theorem; permutation groups, permutations as products of cycles, even and odd permutations, normal subgroups, quotient groups; isomorphism theorems, correspondence theorem;

Unit II: Sylow's Theorem

Group action; Cayley's theorem, group of symmetries, dihedral groups and their elementary properties; orbit decomposition; counting formula; class equation, consequences for p-groups; Sylow's theorems (proofs using group actions). Applications of Sylow's theorems, conjugacy classes in S_n and A_n, simplicity of A_n. Direct product; structure theorem for finite abelian groups; invariants of a finite abelian group (Statements only)

Unit III: Rings

Basic properties and examples of ring, domain, division ring and field; direct products of rings; characteristic of a domain; field of fractions of an integral domain; ring homomorphisms (always unitary); ideals; factor rings; prime and maximal ideals, principal ideal domain; Euclidean domain; unique factorization domain.

Unit IV: Polynomial Rings

A brief review of polynomial rings over a field; reducible and irreducible polynomials, Gauss' theorem for reducibility of $f(x) \in Z[x]$; Eisenstein's criterion for irreducibility of $f(x) \in Z[x]$ over Q, roots of polynomials; finite fields of orders 4, 8, 9 and 27 using irreducible polynomials over Z_2 and Z_3.

Textbooks:

1. Bhattacharya,P.B., Jain, S. K. and Nagpal S. R. (2000) *Basic Abstract Algebra* (3rd edition), Cambridge University Press.
2. Jacobson, N. (2002) *Basic Algebra I* (3rd edition), Hindustan Publishing Corporation, New Delhi.
3. Gallian, J. A. (1999) *Gallian Contemporary Abstract Algebra* (4th edition), Narosa Publishing House, New Delhi.

Reference books:

1. Herstein, I. N. (2003) *Topics in Algebra* (4th edition), Wiley Eastern Limited, New Delhi.
2. Fraleigh, J. B. (2002) *A First Course in Abstract Algebra* (4th edition), Narosa Publishing House, New Delhi.
3. Dummit, D.S. and Foote, R.M (2003) *Abstract Algebra*, John Wiley & Sons.

MTH-PG-C204A: NUMERICAL ANALYSIS

Credit: 4

Unit I: Non- linear system of equations and linear system of algebraic equations

System of non-linear equations: Fixed point iteration method for the system $x = g(x)$, sufficient condition for convergence, Newton's method for nonlinear systems

Solution of Linear equations: Direct methods: Gaussian elimination, LU and Cholesky factorizations. Operational counts for all these direct methods. Iterative methods: General framework for iterative methods, Jacobi and Gauss Seidel methods, Necessary and sufficient conditions for convergence, order of convergence, successive relaxation method.

Unit II: Eigenvalue Problem

Gershgorin theorem, Power and inverse power method, QR method. Jacobi, Givens and Householder's methods for symmetric eigenvalue problem

Unit III: Finite Difference Methods

Finite difference methods for two point boundary value problems, convergence and stability. Finite difference methods for parabolic, hyperbolic and elliptic partial differential equations: Discretization error, Idea of convergence and stability, Explicit and Crank-Nicolson implicit method of solution of one dimensional heat conduction equation: convergence and stability. Standard and diagonal five point formula for solving Laplace and Poisson equations, Explicit and Implicit method of solving Cauchy problem of one-dimensional wave equation, CFL conditions of stability and convergence, Finite difference approximations in polar coordinates.

Unit IV: Practical

1. Gauss-Jordan method
2. LU and Cholesky factorization methods
3. Inverse of a matrix
4. S.O.R. / S.U.R. method
5. Relaxation method
6. Power and inverse power methods
7. Jacobi, Givens and Householder's methods
8. Solution of one dimensional heat conduction equation by
    i) Explicit and
    ii) Crank-Nicolson implicit method
9. Solution of Laplace equation
10. Solution of Poisson equation
11. Solution of one-dimensional wave equation

Textbooks:

1. K. E. Atkinson, K. E. (1989) *Introduction to Numerical Analysis*, John Wiley.
2. Smith, G. D. (1986) *Numerical Solution of Partial Differential Equations*, Oxford University Press.

MTH-PG-C204B: CRYPTOGRAPHY

Credit: 4

Unit I: Number Theory and Time estimates required for Cryptography
The big Oh notation, time estimates for doing addition, subtraction, multiplication, division. Euclidean Algorithm and the time estimate to find the greatest common divisor of two integers, extended Euclidean algorithm. Properties of congruences: addition, multiplication, subtraction and division; solution of linear congruences, modular exponentiation by repeated squaring method.

Unit II: Fundamental Theorems
Fermat's little theorem, Euler's totient function, Euler's theorem, Primitive roots. Finite fields: Primitive polynomials, Irreducible polynomials, Time estimations for doing arithmetic operations in finite fields, Construction of finite fields.

Unit III: Classical Cryptosystems
Shift cipher, Affine cipher, Substitution cipher, Vigenere cipher, Hill cipher, permutation cipher. Public Key cryptography: One way function, Trap door functions, Concept of public key cryptography, RSA, Digital signature scheme.

Unit IV: Primality Testing and Integer Factorization
Primality testing: pseudo primes, Rabin Miller probabilistic primality test, Carmichael numbers. Factoring algorithms: Pollard's rho method, Pollard's p-1 method, Fermat's factorization method.Discrete logarithm, Diffie-Hellman Key exchange protocol, El Gamal cryptosystem over prime field and finite fields, El Gamal digital signature scheme.

(Note: A basic introduction to Elliptic curve cryptography should be taught for the benefit of the students but it should not be included for examination purpose).

Text book:

1. Koblitz, N. (1994) *A course in Number Theory and Cryptography*, (Second Ed.), Springer-Verlag.

Reference books:

1. Stinson, D. R. (1995) *Cryptography: Theory and Practice*, CRC Press series on Discrete Mathematics and its applications.
2. Yan, S. Y. (2003) *Primality Testing and Integer Factorization in Public-Key Cryptography*, Springer

MTH-PG-C301: TOPOLOGY

Credit: 4

Unit I: Topology
Definition and examples of topological spaces; basis and sub basis; order topology; subspace topology.Continuity and related concepts; product topology; quotient topology; countability axioms; Lindelof spaces and separable spaces.

Unit II: Connectedness
Connected spaces, generation of connected sets; component, path component; local connectedness, local path-connectedness.

Unit III: Compactness
Compact spaces; limit point compact and sequentially compact spaces; locally compact spaces; one point compactification; finite product of compact spaces, statement of Tychonoff's theorem (Proof of finite product only).

Unit IV: Separability & Countability
Separation axioms; Urysohn’s lemma; Tietze’s extension theorem; Urysohn’s embedding lemma and Urysohn’s metrization theorem for second countable spaces.

Textbooks:

1. Munkres, J. R. (2000) *Topology: a First Course*, Prentice-Hall of India Ltd., New Delhi.

Reference books:

1. J. Dugundji (1990) *General Topology*, Universal Book Stall, New Delhi.
2. Pervin, W. J. (1964) *Foundations of General Topology*, Academic Press, New York.
3. Willard, S. (1970) *General Topology*, Addison-Wesley Publishing Company, Massachusetts.
4. Armstrong, M. A. (2005) *Basic Topology*, Springer International Ed.
5. Kelley, J. L. (1990) *General Topology*, Springer Verlag, New York.
6. Joshi, K. D. (2002) *An Introduction to General Topology* (2nd edition), Wiley Eastern Ltd., New Delhi.

MTH-PG-C302: FIELD THEORY

Credit: 4

Unit I: Field Theory
Extension fields, finite extensions; algebraic and transcendental elements, adjunction of algebraic elements, Kronecker theorem, algebraic extensions, splitting fields – existence and uniqueness; extension of base field isomorphism to splitting fields;

Unit II: Polynomials
Simple and multiple roots of polynomials, criterion for simple roots, separable and inseparable polynomials; perfect fields; separable and inseparable extensions, finite fields; prime fields and their relation to splitting fields; Frobenius endomorphisms; roots of unity and cyclotomic polynomials.

Unit III: Galois group
Algebraically closed fields and algebraic closures, primitive element theorem; normal extensions; automorphism groups and fixed fields; Galois pairing; determination of Galois groups, fundamental theorem of Galois theory, abelian and cyclic extensions.

Unit IV: Solvability
Normal and subnormal series, composition series, Jordan-Holder theorem (statement only); solvable groups, Sovability by radicals; solvability of algebraic equations; symmetric functions; ruler and compass constructions, fundamental theorem of algebra.

Textbooks:

1. T. I. F. R. *Mathematical pamphlets*, No. 3, (1965) Galois Theory.
2. Artin, E. (1997) *Galois Theory*, Edited by Arthur N. Milgram, Dover Publications.

Reference books:

1. Herstein, I. N. (2003) *Topics in Algebra* (4th edition), Wiley Eastern Limited, New Delhi.
2. Bhattacharya, P. B., Jain, S. K. and Nagpal, S. R. (2000) *Basic Abstract Algebra* (3rd edition), Cambridge University Press.
3. Jacobson, N. (2002) *Basic Algebra I* (3rd edition), Hindustan Publishing Corporation, New Delhi.
4. Fraleigh, J. B. (2002) *A First Course in Abstract Algebra* (4th edition), Narosa Publishing House, New Delhi.

MTH-PG-C303: MEASURE THEORY

Credit: 4

Unit I: Measures
Algebras, sigma-algebras, monotone classes; outer measures and Caratheodory's extension theorem; existence of Lebesgue measure and of non-measurble sets.

Unit II: Integration
Measurable functions, monotone approximability by simple functions, integrability and Lebesgue integration; standard limit theorems: Fatou's lemma, monotone convergence and dominated convergence theorems; almost everywhere considerations.

Unit III: Random Variables & Distributions
Probability, random variables and their distributions, joint distributions and independence, Borel-Cantelli lemma and Kolmogorov's zero-one law, Some of the more standard distributions - both discrete (Bernouilli, Binomial, Poisson, etc.) and continuous (Uniform, Normal, etc.); a brief introduction to conditional expectations and probabilities.

Unit IV: Measures on Product Spaces
Product measures, theorems of Tonelli and Fubini, independence and product measures, infinite products and finite state Markov Chains, Kolmogorov consistency theorem. Characteristic funcions, modes of convergence.

Text book:

1. Athreya, S. R. and Sunder, V.S. (2008) *Probability and Measure*, Universities Press, India.

Reference:

1. Rana, I.K. (2002) *An Introduction to Measure and Integration*, American Math. Soc.
2. Chung, k. L. (2001) *A Course in Probability Theory*, Academic Press.

MTH-PG-C304A: OPTIMIZATION TECHNIQUE

Credit: 4

Unit I: Introduction

Nature and Features of Operations Research (O.R)- Convex set- Polyhedral Convex Set- Linear Programming (L.P)-Mathematical Formulation of the Problem- Graphical Solution Method-Some Exceptional Cases-General Linear Programming Problem (General L.P.P)

Unit II: Linear Programming Problem

Slack and Surplus Variables-Reformulation of the General L.P.P.- Simplex Method- Matrix Notation-Duality (Statement only of Property without Proof)- Initial Simplex Tableau- Pivot-Calculating the new Simplex Tableau-Terminal Simplex Tableau- Algorithm of the Simplex Method.

Unit III: Games and Strategies

Introduction- Two- person Zero-sum games-Pay-off Matrix – some basic terms-the Maximum –Minimal Principle-Theorem on Maximum and Minimal Values of the Game-Saddle Point and Value of the Game-Rule for determining a Saddle Point-Games without Saddle Points-Mixed Strategies-Graphic solution of 2 x n and m x 2 games- Dominance Property- General rule for Dominance-Modified Dominance Property.

Unit IV: Integer Programming

Travelling Salesman Problem, Transport and Assignment Problem, Max flow-Min cut problem, Minimal spanning tree, shortest path problem.

Text Book:

1. Hadley, G. (1966) *Linear Programming*, Addison.
2. Gale, D. (1989) *The Theory of Linear Economic Model*, University of Chicago Press.
3. Swarup, K, Gupta, P. K. and Mohan, M. (2002) *Operations Research*, Sultan Chand & Sons, New Delhi.

Reference Books:

1. Friderick S. H. and Gerald J. L. (1974) *Operations Research*, Holden-Day Inc, San Fransisco.
2. Hamdy A. T. (2002) *Operation Research: An Introduction*, Prentice-Hall of India Pvt. Ltd., New Delhi.

MTH-PG-C304B: NON-LINEAR DYNAMICS

Credit: 4

Unit I: Linear Systems
System of linear ordinary differential equations; Fundamentals of linear systems; Linear Systems in R^2; Stability theory, phase portraits in R^2.

Unit II: Non-Linear Systems
System of nonlinear ordinary differential equations; Fundamental existence-uniqueness theorem, dependence on initial conditions and parameters; flow defined by differential equations; linearization; Hartman-Grobman theorem; Stability and Lyapunov functions.

Unit III: Stable Points
Stable manifold theorem, Center manifold theorem; Elementary bifurcations- Saddle-node, Transcritical, Pitch-fork, Hopf bifurcation.

Unit IV: Attractors
Limit sets and attractors, periodic orbits and limit cycles, stable manifold theorem for periodic orbits, Lienard systems, Bendixon's Criteria, global bifurcation of systems in R^2 .

Text Books

1. Perko, L. (2001) *Differential Equations and Dynamical Systems*, Springer.
2. Jordan, D. W. and Smith, P. (1999) *Nonlinear Ordinary Differential Equations: An Introduction to Dynamical Systems*, Oxford University Press.

Departmental Electives:

1. Algebraic Topology
2. Commutative Algebra
3. Algebraic Geometry
4. Differential Geometry
5. Operator theory
6. Functional Analysis

1. **ALGEBRAIC TOPOLOGY**

Credit: 4

Unit I: The fundamental group

Homotopy of paths, fundamental group of a topological space, fundamental group functor, homotopy of maps of topological spaces; homotopy equivalence; contractible and simply connected spaces; Calculation of fundamental groups of $\mathbb{S}^n$ (n > 1) using Van Kampen's theorem (special case); fundamental group of a topological group; Brouwer's fixed point theorem; fundamental theorem of algebra; vector fields, Frobenius theorem on eigenvalues of 3 x 3 matrices.

Unit II: Covering spaces

Covering spaces, unique lifting theorem, path-lifting theorem, covering homotopy theorem, fundamental group of $\mathbb{S}^1$, $\mathbb{S}^1 \times \mathbb{S}^1$ etc., degree of maps of $\mathbb{S}^1$ and applications; criterion of lifting of maps in terms of fundamental groups; universal coverings and its existence; special cases of manifolds and topological groups.

Unit III: Homology

Category and Functors, Singular homology, relative homology, Eilenberg-Steenrod axioms (without proof), Reduced homology, relation between Π_1 and H_1;.

Unit IV: Applications

Calculations of homology of $\mathbb{S}^n$; Brouwer's fixed point theorem and its applications to spheres and vector fields; Meyer-Vietoris sequence and its application.

Textbooks:

1. Munkres, J. R. (2000) *Topology: A First Course*, Prentice-Hall of India Ltd., New Delhi.
2. Greenberg, M. J. and Harper, J. R. (1997) *Algebraic Topology: A First Course* (2nd edition), Addison-Wesley Publishing Co.
3. Hatcher, A. (2002) *Algebraic Topology*, Cambridge University Press.

Reference:

1. Armstrong , M. A. (2000) *Basic Topology*, UTM Spinger
2. Greenberg, M. J. and Harper, J. R. (1997) *Algebraic Topology: A First Course* (2nd edition), Addison-Wesley Publishing Co.
3. Hatcher, A. (2002) *Algebraic Topology*, Cambridge University Press.
4. Spanier, E. H. (2000) *Algebraic Topology* (2nd edition), Springer-Verlag, New York.
5. Rotman, J. J. (2004) *An Introduction to Algebraic Topology, Text in Mathematics*, No. 119, Springer, New York.

2. COMMUTATIVE ALGEBRA

Credit: 4

Unit I: Rings and Ideals
A brief review of rings, ideals and homomorphisms, Operations on ideals, Extension and contraction of ideals, Nil radical and Jacobson radical.

Unit II: Modules
Modules, sub modules, homomorphism, direct sum and products of modules, exact sequences Tensor product of modules and algebras and basic properties

Unit III: Modules of Fractions and Primary Decomposition
Rings and modules of fractions, Primary decomposition,

Unit IV: Integral Dependence and Valuation Rings
Integral dependence, Going up and going down theorems, Valuation rings; Noetherian rings, Artin rings

Text Books:
1. M. F. Atiyah & I. G. Macdonald, *Introduction to Commutative Rings*, Addison Wesley
2. Zarinski and P. Samuel, *Commutative Algebra with a view towards Algebraic Geometry*, Springer

Reference Books:
1. Irving Kapalansky– *Commutative Rings*
2. N. S. Gopalakrishnan – *Commutative Algebra*, Oxonian Press

3. ALGEBRAIC GEOMETRY

Credit: 4

Unit I: Affine algebraic sets
Affine spaces and algebraic sets, Noetherian rings, Hilbert basis theorem, affine algebraic sets as finite intersection of hypersurfaces; Ideal of a set of points, co-ordinate ring, morphism between algebraic sets, isomorphism. Integral extensions, Noether's normalization lemma

Unit II: Hilbert's Nullstellensatz and applications
Correspondence between radical ideals and algebraic sets, prime ideals and irreducible algebraic sets, maximal ideals and points, contrapositive equivalence between affine algebras with algebra homomorphisms and algebraic sets with morphisms, between affine domains and irreducible algebraic sets, decomposition of an algebraic set into irreducible components. Zariski topology on affine spaces, algebraic subsets of the pl ane.

Unit III: Projective spaces
Homogeneous co-ordinates, hyperplane at infinity, projective algebraic sets, homogeneous id eals and projective Nullstellensatz; Zariski topology on projective spaces. Twisted cubic in P _3(k).
Local properties of plane curves: multiple points and tangent lines, multiplicity and local rings, intersection numbers; projective plane curves: Linear systems of curves, intersections of projective curves: Bezout's theorem and applications; group structure on a cubic

Unit IV: Introduction to sheaves of affine varieties
Examples of presheaves and sheaves, stalks,
sheafification of a pre-sheaf, sections, structure sheaf, generic stalk and function fields, rational functions and local rings, Affine tangent spaces; Projective varieties and morphisms; Hausdorff axiom.
Prime spectrum of a ring: Zariski topology, structiure aheaf, affine schemes, morphism of affine schemes.
Elementary Dimension Theory, Fibres of a morphism, complete varieties, nonsingularity and regular local rings, Jacobian criterion, non-singular curves and DVR's.

Text Books:

1. W.Fulton - *Algebraic Curves: An introduction to algebraic geometry*
2. C. G. Gibson – *Elementary Geometry of Algebraic Curves*, CUP,
3. D. S. Dummitt and R. M. Foote – *Abstract Algebra*, Wiley, Ch. 15

Reference Books:
1. J. Harris - *Algebraic Geometry, A first course*, Springer
2. M. Reid - *Undergraduate algebraic geometry*, LMS 12, CUP
3. K. Kendig – *Elementary Algebraic Geometry*, Springer
4. D. Mumford – *The Red Book of Varieties and Schemes*, Springer
5. I. R. Shafarevich – *Basic Algebraic Geometry*, Springer

4. DIFFERENTIAL GEOMETRY

Credit: 4

Unit I: Vectors
Vectors in R^3; tangent vectors; tangent spaces; tangent vector fields; derivative mappings; translations; affine transformations and rigid motions (isometries); exterior derivatives.

Unit II: Space Curves
Space curves; arc length; tangent vectors and vector fields on a curve; curvature and torsion; Serret-Frenet formulas; osculating plane; osculating circle; osculating sphere; fundamental theorem of local theory of space curves (existence and uniqueness theorems).

Unit III: Surfaces
Surfaces and their (local) parametrization on coordinate systems; change of parameters; parametrized surfaces; curves on surfaces; tangent and normal vectors; tangent and normal

vector fields on a surface; first, second and third fundamental forms of a surface at a point; Gauss mapping.

Unit IV: Curvature
Normal sections and normal curvature of a surface at a point; Meusnier's theorem; elliptic, hyperbolic, parabolic and planar points; Dupin indicatrix; principal directions; principal curvatures of a surface at a point; Mean curvature and Gaussian curvature of a surface at a point.
Line of curvature; asymptotic curves; conjugate directions; fundamental equations of the local theory of surfaces; statement of Bonnet's fundamental theorem of local theory of surfaces.

Textbook:

1. Hsiung, C. C. (1997) *A first Course in Differential Geometry*, International Press, University of Michigan.

Reference:

1. Eissenhart, P. (1960) *A Treatise on the Differential Geometry of Curves and Surfaces*, Dover Publications, Inc., New York.
2. Weatherburn, C. R. (1964) *Differential Geometry of Three Dimensions*, The English Language Book Society and Cambridge University Press.
3. Willmore, T. S. (1979) *An Introduction to Differential Geometry*, Clarendon Press, Oxford.
4. Klingenberg, V. (1978) *A Course in Differential Geometry*, Graduate Texts in Mathematics 51, Springer-Verlag.
5. Pressley, A. (2005) *Elementary Differential Geometry*, Springer International Edition.

5. OPERATOR THEORY

Credit: 4

Unit I: Compact operators on Hilbert spaces
Fredholm theory, index

Unit II: C* algebras
Non-communicative states and representations, Gelfand- Neumark representation theorem

Unit III: Von-Neumann Algebras
Projections, Double communtant theorem

Unit IV: L^∞ functional Calculus
Toeplitz operators

Text Books:

1. W. Arveson- *An Invitation to C* algebras*, GTM(39), Springer-Verlag
2. V.S. Sunder- *An invitation to von Neuman algebras*, Springer-Verlag

References:

1. N.Dunford and J.T. Schwarz- *Linear Operators*, Part-II: spectral theoery.

Self adjoint operators in Hilbert space, John Wiley.

6. FUNCTIONAL ANALYSIS

Credit: 4

Unit I: Normed Linear Spaces and Banach Spaces
Bounded Linear Operatores, Duals, Hahn-Banach theorem, Uniform boundedness principle.

Unit II: Open mappings and closed graph theorems
Some applications, dual spaces, computing duals of $L^p, (1 \leq p < \infty)$ and C[0, 1], reflexive spaces;

Unit III: Weak and weak* topologies
Banach Alaoglu theorem, Hilbert Spaces-orthogonal sets, projection theorem, Riesz representation theorem

Unit IV: Adjoint operator
Self-adjoint, normal and unitary operators; Projections, spectrum and spectral radius, spectral theorem for compact operators

Text Book:

1. G.F. Simmons- *Topology and Modern Analysis* (Ch. 9, 10, 11,12), TMH
2. J.B. Conway- *A first course in Functional Analysis*, Springer

Reference Books:

W. Rudin- *Real and Complex Analysis*, TMH